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Stuart W. Altizer, John S. Asvestas and James M. Stamm

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SCATTERING BY ELECTRICALLY LARGE OBJECTS WITH CAVITY-LIKE FEATURES.

A HYBRID APPROACH.

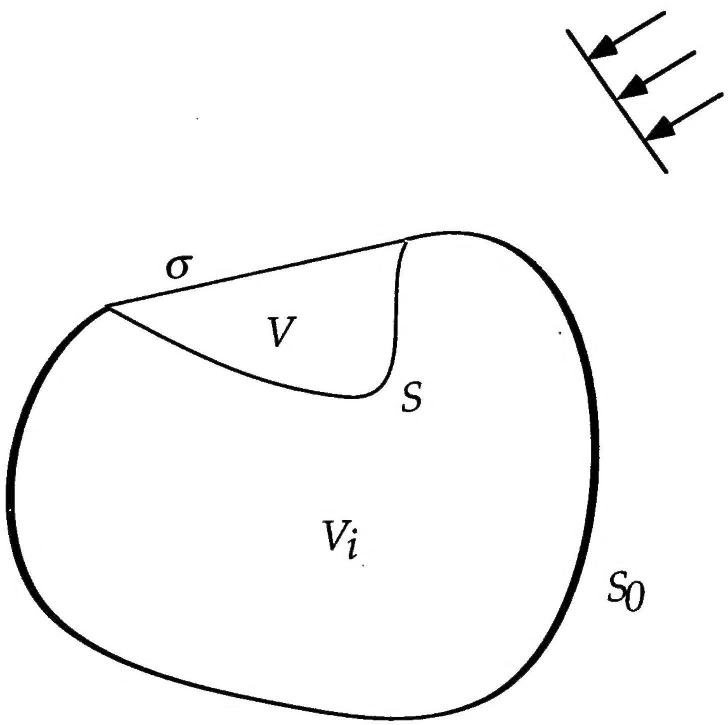
by.

Stuart W. Altizer, John S. Asvestas and James M. Stamm.

RF Sensors Branch.

Naval Air Warfare Center – Aircraft Division.

THE PROBLEM: LARGE SCATTERER WITH A CAVITY



Incident
plane wave propagating
in free space.
Harmonic ($\exp[-i\omega t]$)
time dependence.
Incident fields: $\mathbf{E}^i$, $\mathbf{H}^i$

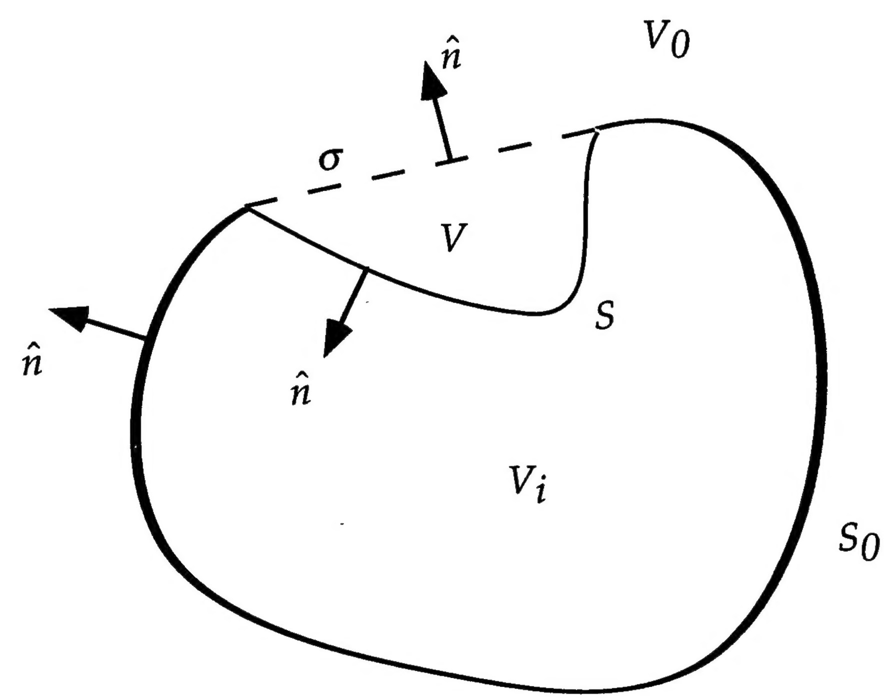
Scattered fields: $\mathbf{E}^s$, $\mathbf{H}^s$

Total fields in exterior:
$\mathbf{E}^t = \mathbf{E}^i + \mathbf{E}^s$
$\mathbf{H}^t = \mathbf{H}^i + \mathbf{H}^s$

Total fields in V : $\mathbf{E}$, $\mathbf{H}$

The closed surface $S \cup S_0$
is perfectly conducting.
Constitutive parameters in V
are arbitrary.

ANOTHER LOOK AT THE GEOMETRY

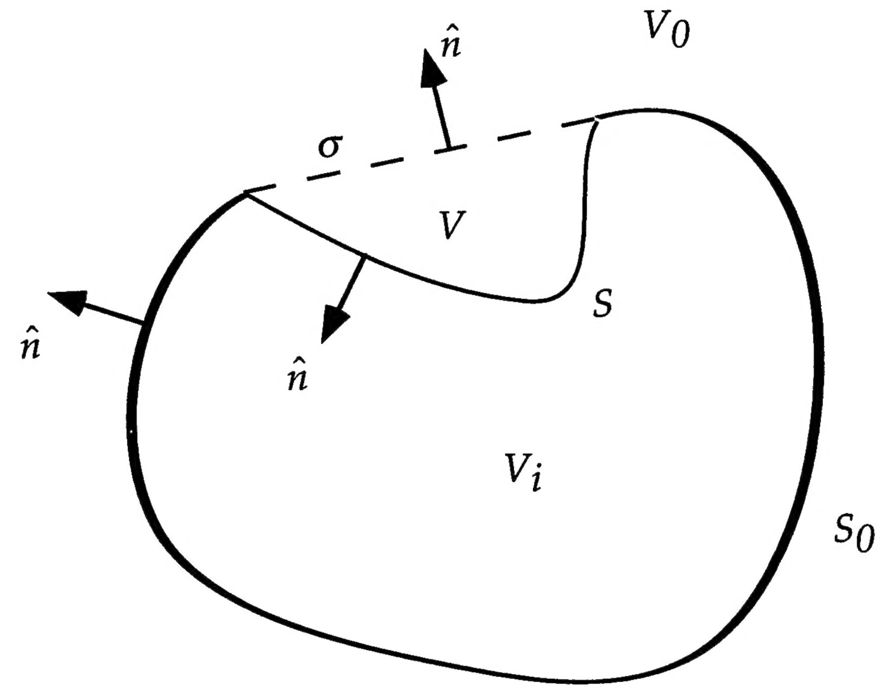


- CAVITY SIZE EXAGGERATED
- SURFACE $S \cup S_0$ IS VERY MANY SQUARE WAVELENGTHS IN AREA

⇨ A HYBRID METHOD IS CALLED FOR.

SPECIFICALLY

- USE A HIGH-FREQUENCY METHOD FOR THE LARGE PART OF THE STRUCTURE (S_0)
- USE THE METHOD OF MOMENTS IN THE CAVITY (V)
- COUPLE THE TWO AT THE ENTRANCE, σ, *TO THE CAVITY*



MOST RECENT HYBRID APPROACH TO THIS PROBLEM

Jin, J.-M., Ni, S.S. and Lee, S.W., "Hybridization of SBR and FEM for scattering by large bodies with cracks and cavities", *IEEE Trans. Antennas Propagat.*, Vol. 43 (10), pp. 1130-1139, Oct. 1995

SBR: Shooting and bouncing rays; FEM: Finite element method

PRINCIPAL DIFFERENCE BETWEEN THIS METHOD AND THE ONE WE WILL DESCRIBE IS IN THE KIND OF GREEN'S FUNCTION (GF) THEY USE:

•THE SBR/FEM USES A SPECIALIZED GF

•OUR METHOD USES THE FREE SPACE GF

FREE SPACE DYADIC GREEN'S FUNCTION

$$\underline{\mathbf{\Gamma}}(k;\mathbf{r},\mathbf{r}') = ik\nabla\times[G(k;\mathbf{r},\mathbf{r}')\underline{\mathbf{I}}] \quad , \quad G(k;\mathbf{r},\mathbf{r}') = -\,\mathrm{e}^{ikR}/4\pi R \quad , \quad R = |\mathbf{r}-\mathbf{r}'|$$

- IT SATISFIES

$$\nabla\times\nabla\times\underline{\mathbf{\Gamma}}(k;\mathbf{r},\mathbf{r}') - k^2\underline{\mathbf{\Gamma}}(k;\mathbf{r},\mathbf{r}') = -ik\nabla\times[\delta(\mathbf{r},\mathbf{r}')\underline{\mathbf{I}}]$$

- AND CAN BE INTERPRETED AS

  - THE MAGNETIC FIELD OF THREE ORTHOGONALLY CROSSED ELECTRIC DIPOLES

  OR

  - AS THE ELECTRIC FIELD OF THREE ORTHOGONALLY CROSSED MAGNETIC DIPOLES

- AND IT IS VERY EASY TO COMPUTE

DYADICS OF THE FIRST AND SECOND KIND

•USING $\underline{\mathbf{\Gamma}}$ WE CAN CONSTRUCT DYADICS $\underline{\mathbf{\Gamma}}_1$ AND $\underline{\mathbf{\Gamma}}_2$ THAT SATISFY

$$\hat{n}\times\underline{\mathbf{\Gamma}}_1(k;\mathbf{r},\mathbf{r}')=0 \quad , \quad \hat{n}\times\nabla\times\underline{\mathbf{\Gamma}}_2(k;\mathbf{r},\mathbf{r}')=0$$

ON SOME SURFACE S.

•IF S IS PERFECTLY CONDUCTING, THEN $\underline{\mathbf{\Gamma}}_1$ IS AN ELECTRIC FIELD WHILE $\underline{\mathbf{\Gamma}}_2$ IS A MAGNETIC FIELD.

•WE CAN WRITE FOR THESE DYADICS

$$\underline{\mathbf{\Gamma}}_1=\underline{\mathbf{\Gamma}}+\underline{\mathbf{\Gamma}}_1^S \text{ AND } \underline{\mathbf{\Gamma}}_2=\underline{\mathbf{\Gamma}}+\underline{\mathbf{\Gamma}}_2^S$$

•THUS TO DETERMINE THESE DYADICS WE MUST FIND THE FIELDS OF INFINITESIMAL DIPOLES IN THE PRESENCE OF A PERFECT CONDUCTOR.
•THIS IS A MORE DIFFICULT UNDERTAKING THAN THE ORIGINAL PROBLEM IF THE SCATTERER IS GEOMETRICALLY COMPLEX.

•FOR THIS REASON WE PREFER AN APPROACH THAT USES THE *FREE-SPACE* GREEN'S FUNCTION.

PROPOSED METHOD

- THE MAGNETIC FIELD IN V_0

$$\mathbf{H}^s(\mathbf{r}') = \frac{Y_0}{k_0^2} \int_\sigma \left\{ \left[\hat{n} \times \mathbf{E}^t(\mathbf{r})\right] \cdot \nabla \times \underline{\mathbf{\Gamma}}(k_0;\mathbf{r},\mathbf{r}') + ik_0 Z_0 \left[\hat{n} \times \mathbf{H}^t(\mathbf{r})\right] \cdot \underline{\mathbf{\Gamma}}(k_0;\mathbf{r},\mathbf{r}') \right\} dS$$

$$+ \frac{i}{k_0} \int_{S_0} \left[\hat{n} \times \mathbf{H}^t(\mathbf{r})\right] \cdot \underline{\mathbf{\Gamma}}(k_0;\mathbf{r},\mathbf{r}')\, dS \quad , \quad \mathbf{r}' \in V_0$$

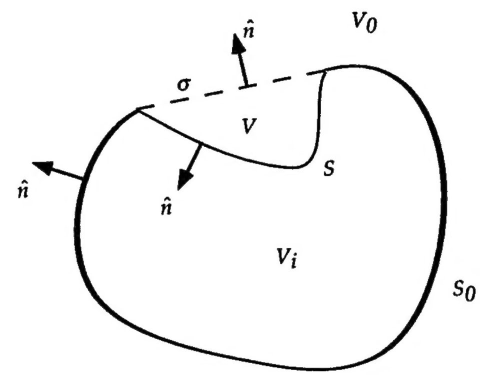
- LET

$$\mathbf{J}^t_{S_0}(\mathbf{r}) = \hat{n} \times \mathbf{H}^t(\mathbf{r}) \,,\, \mathbf{r} \in S_0 \quad ; \quad \mathbf{M}^t_\sigma(\mathbf{r}) = -\hat{n} \times \mathbf{E}^t(\mathbf{r}) \,,\, \mathbf{J}^t_\sigma(\mathbf{r}) = \hat{n} \times \mathbf{H}^t(\mathbf{r}) \,,\, \mathbf{r} \in \sigma$$

- THEN

$$\mathbf{H}^t(\mathbf{r}') = \mathbf{H}^{inc}(\mathbf{r}') + \frac{Y_0}{k_0^2} \int_\sigma \left\{ -\mathbf{M}^t_\sigma(\mathbf{r}) \cdot \nabla \times \underline{\mathbf{\Gamma}}(k_0;\mathbf{r},\mathbf{r}') + ik_0 Z_0 \mathbf{J}^t_\sigma(\mathbf{r}) \cdot \underline{\mathbf{\Gamma}}(k_0;\mathbf{r},\mathbf{r}') \right\} dS$$

$$+ \frac{i}{k_0} \int_{S_0} \mathbf{J}^t_{S_0}(\mathbf{r}) \cdot \underline{\mathbf{\Gamma}}(k_0;\mathbf{r},\mathbf{r}')\, dS \quad , \quad \mathbf{r}' \in V_0$$



- MAKE A HIGH-FREQUENCY APPROXIMATION TO THE SURFACE CURRENT ON S_0

$$\mathbf{H}_0^t(\mathbf{r}') = \frac{i}{k_0} \int_{S_0^*} \mathbf{J}_{S_0}^t(\mathbf{r}) \cdot \underline{\mathbf{\Gamma}}(k_0; \mathbf{r}, \mathbf{r}')\, dS \quad , \quad \mathbf{r}' \in V_0$$

$$\mathbf{H}^{kn}(\mathbf{r}') = \mathbf{H}^{inc}(\mathbf{r}') + \mathbf{H}_0^t(\mathbf{r}') \quad , \quad \mathbf{r}' \in V_0$$

- THEN

$$\mathbf{H}^t(\mathbf{r}') = \mathbf{H}^{kn}(\mathbf{r}') + \frac{Y_0}{k_0^2} \int_\sigma \left\{ -\mathbf{M}_\sigma^t(\mathbf{r}) \cdot \nabla \times \underline{\mathbf{\Gamma}}(k_0; \mathbf{r}, \mathbf{r}') + ik_0 Z_0 \mathbf{J}_\sigma^t(\mathbf{r}) \cdot \underline{\mathbf{\Gamma}}(k_0; \mathbf{r}, \mathbf{r}') \right\} dS \,,\ \mathbf{r}' \in V_0$$

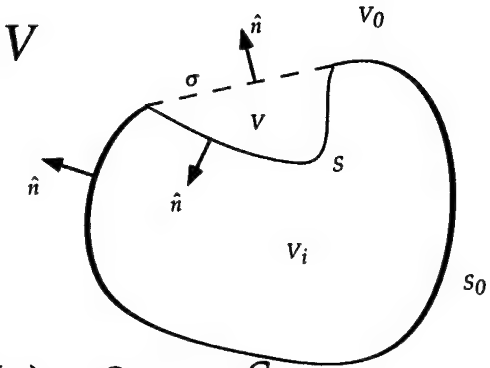
- SIMILARLY

$$\mathbf{E}^t(\mathbf{r}') = \mathbf{E}^{kn}(\mathbf{r}') - \frac{Z_0}{k_0^2} \int_\sigma \left\{ \mathbf{J}_\sigma^t(\mathbf{r}) \cdot \nabla \times \underline{\mathbf{\Gamma}}(k_0; \mathbf{r}, \mathbf{r}') + ik_0 Y_0 \mathbf{M}_\sigma^t(\mathbf{r}) \cdot \underline{\mathbf{\Gamma}}(k_0; \mathbf{r}, \mathbf{r}') \right\} dS \,,\ \mathbf{r}' \in V_0$$

- WITH

$$\mathbf{E}^{kn}(\mathbf{r}') = \mathbf{E}^{inc}(\mathbf{r}') + \mathbf{E}_0^t(\mathbf{r}'), \quad \mathbf{E}_0^t(\mathbf{r}') = -\frac{Z_0}{k_0^2} \int_{S_0^*} \mathbf{J}_{S_0}^t(\mathbf{r}) \cdot \nabla \times \underline{\mathbf{\Gamma}}(k_0; \mathbf{r}, \mathbf{r}')\, dS \,,\ \mathbf{r}' \in V_0$$

INTEGRAL REPRESENTATIONS IN V



- WITH $\quad \mathbf{J}_S(\mathbf{r}) = -\hat{n} \times \mathbf{H}(\mathbf{r}) \quad , \quad \mathbf{r} \in S$

- AND THE TRANSMISSION AND BOUNDARY CONDITIONS

$$\mathbf{M}_\sigma^t(\mathbf{r}) = -\hat{n} \times \mathbf{E}(\mathbf{r}), \quad \mathbf{J}_\sigma^t(\mathbf{r}) = \hat{n} \times \mathbf{H}(\mathbf{r}), \quad \mathbf{r} \in \sigma \; ; \quad \hat{n} \times \mathbf{E}(\mathbf{r}) = \mathbf{0} \, , \mathbf{r} \in S$$

- WE GET

$$\mathbf{H}(\mathbf{r}') = -\frac{Y_1}{k_1{}^2} \int_\sigma \left\{ -\mathbf{M}_\sigma^t(\mathbf{r}) \cdot \nabla \times \underline{\mathbf{\Gamma}}(k_1; \mathbf{r}, \mathbf{r}') + i k_1 Z_1 \mathbf{J}_\sigma^t(\mathbf{r}) \cdot \underline{\mathbf{\Gamma}}(k_1; \mathbf{r}, \mathbf{r}') \right\} dS$$

$$+ \frac{i}{k_1} \int_S \mathbf{J}_S(\mathbf{r}) \cdot \underline{\mathbf{\Gamma}}(k_1; \mathbf{r}, \mathbf{r}') \, dS \quad , \quad \mathbf{r}' \in V$$

$$\mathbf{E}(\mathbf{r}') = \frac{Z_1}{k_1{}^2} \int_\sigma \left\{ \mathbf{J}_\sigma^t(\mathbf{r}) \cdot \nabla \times \underline{\mathbf{\Gamma}}(k_1; \mathbf{r}, \mathbf{r}') + i k_1 Y_1 \mathbf{M}_\sigma^t(\mathbf{r}) \cdot \underline{\mathbf{\Gamma}}(k_1; \mathbf{r}, \mathbf{r}') \right\} dS$$

$$- \frac{Z_1}{k_1{}^2} \int_S \mathbf{J}_S(\mathbf{r}) \cdot \nabla \times \underline{\mathbf{\Gamma}}(k_1; \mathbf{r}, \mathbf{r}') \, dS \quad , \quad \mathbf{r}' \in V$$

FROM INTEGRAL REPRESENTATIONS TO INTEGRAL EQUATIONS

- LET $\mathbf{r}'$ APPROACH A POINT ON σ FROM V_0

$$\frac{1}{2}\mathbf{J}_\sigma^t(\mathbf{r}') = \mathbf{J}_\sigma^{kn}(\mathbf{r}') + \frac{Y_0}{k_0{}^2}\hat{n}' \times \int_\sigma \left\{ -\mathbf{M}_\sigma^t(\mathbf{r}) \cdot \nabla \times \underline{\mathbf{\Gamma}}(k_0;\mathbf{r},\mathbf{r}') + ik_0 Z_0 \mathbf{J}_\sigma^t(\mathbf{r}) \cdot \underline{\mathbf{\Gamma}}(k_0;\mathbf{r},\mathbf{r}') \right\} dS \ , \ \mathbf{r}' \in \sigma$$

$$\frac{1}{2}\mathbf{M}_\sigma^t(\mathbf{r}') = \mathbf{M}_\sigma^{kn}(\mathbf{r}') + \frac{Z_0}{k_0{}^2}\hat{n}' \times \int_\sigma \left\{ \mathbf{J}_\sigma^t(\mathbf{r}) \cdot \nabla \times \underline{\mathbf{\Gamma}}(k_0;\mathbf{r},\mathbf{r}') + ik_0 Y_0 \mathbf{M}_\sigma^t(\mathbf{r}) \cdot \underline{\mathbf{\Gamma}}(k_0;\mathbf{r},\mathbf{r}') \right\} dS \ , \ \mathbf{r}' \in \sigma$$

- WHERE

$$\mathbf{M}_\sigma^{kn}(\mathbf{r}) = -\hat{n} \times \mathbf{E}^{kn}(\mathbf{r}) \quad , \quad \mathbf{J}_\sigma^{kn}(\mathbf{r}) = \hat{n} \times \mathbf{H}^{kn}(\mathbf{r}) \quad , \quad \mathbf{r} \in \sigma$$

•LET **r'** APPROACH A POINT ON σ FROM V_i

$$\frac{1}{2}\mathbf{J}_\sigma^t(\mathbf{r}') = -\frac{Y_1}{k_1^2}\hat{n}' \times \int_\sigma \left\{ -\mathbf{M}_\sigma^t(\mathbf{r}) \cdot \nabla \times \underline{\mathbf{\Gamma}}(k_1;\mathbf{r},\mathbf{r}') + ik_1 Z_1 \mathbf{J}_\sigma^t(\mathbf{r}) \cdot \underline{\mathbf{\Gamma}}(k_1;\mathbf{r},\mathbf{r}') \right\} dS$$

$$+\frac{i}{k_1}\hat{n}' \times \int_S \mathbf{J}_S(\mathbf{r}) \cdot \underline{\mathbf{\Gamma}}(k_1;\mathbf{r},\mathbf{r}')\, dS \quad , \quad \mathbf{r}' \in \sigma$$

$$-\frac{1}{2}\mathbf{M}_\sigma^t(\mathbf{r}') = \frac{Z_1}{k_1^2}\hat{n}' \times \int_\sigma \left\{ \mathbf{J}_\sigma^t(\mathbf{r}) \cdot \nabla \times \underline{\mathbf{\Gamma}}(k_1;\mathbf{r},\mathbf{r}') + ik_1 Y_1 \mathbf{M}_\sigma^t(\mathbf{r}) \cdot \underline{\mathbf{\Gamma}}(k_1;\mathbf{r},\mathbf{r}') \right\} dS$$

$$-\frac{Z_1}{k_1^2}\hat{n}' \times \int_S \mathbf{J}_S(\mathbf{r}) \cdot \nabla \times \underline{\mathbf{\Gamma}}(k_1;\mathbf{r},\mathbf{r}')\, dS \quad , \quad \mathbf{r}' \in \sigma$$

•LET **r'** APPROACH A POINT ON S FROM V_i

$$-\frac{1}{2}\mathbf{J}_S(\mathbf{r}') = -\frac{Y_1}{k_1^{\,2}}\hat{n}' \times \int_\sigma \left\{-\mathbf{M}_\sigma^t(\mathbf{r}) \cdot \nabla \times \underline{\mathbf{\Gamma}}(k_1;\mathbf{r},\mathbf{r}') + ik_1 Z_1 \mathbf{J}_\sigma^t(\mathbf{r}) \cdot \underline{\mathbf{\Gamma}}(k_1;\mathbf{r},\mathbf{r}')\right\} dS$$

$$+\frac{i}{k_1}\hat{n}' \times \int_S \mathbf{J}_S(\mathbf{r}) \cdot \underline{\mathbf{\Gamma}}(k_1;\mathbf{r},\mathbf{r}')\, dS \quad , \quad \mathbf{r}' \in S$$

$$\mathbf{0} = \hat{n}' \times \int_\sigma \left\{\mathbf{J}_\sigma^t(\mathbf{r}) \cdot \nabla \times \underline{\mathbf{\Gamma}}(k_1;\mathbf{r},\mathbf{r}') + ik_1 Y_1 \mathbf{M}_\sigma^t(\mathbf{r}) \cdot \underline{\mathbf{\Gamma}}(k_1;\mathbf{r},\mathbf{r}')\right\} dS$$

$$-\hat{n}' \times \int_S \mathbf{J}_S(\mathbf{r}) \cdot \nabla \times \underline{\mathbf{\Gamma}}(k_1;\mathbf{r},\mathbf{r}')\, dS \quad , \quad \mathbf{r}' \in S$$

A SYSTEM OF EQUATIONS

•FOR OBSERVATION POINTS ON S

$$-\frac{1}{2}\mathbf{J}_S(\mathbf{r}') = -\frac{Y_1}{{k_1}^2}\hat{n}' \times \int_\sigma \left\{ -\mathbf{M}_\sigma^t(\mathbf{r}) \cdot \nabla \times \underline{\mathbf{\Gamma}}(k_1;\mathbf{r},\mathbf{r}') + ik_1 Z_1 \mathbf{J}_\sigma^t(\mathbf{r}) \cdot \underline{\mathbf{\Gamma}}(k_1;\mathbf{r},\mathbf{r}') \right\} dS$$

$$+\frac{i}{k_1}\hat{n}' \times \int_S \mathbf{J}_S(\mathbf{r}) \cdot \underline{\mathbf{\Gamma}}(k_1;\mathbf{r},\mathbf{r}')\, dS \quad , \quad \mathbf{r}' \in S$$

•FOR OBSERVATION POINTS ON σ

$$\frac{i}{2}(k_0 Z_0 + k_1 Z_1)\mathbf{J}_\sigma^t(\mathbf{r}') = ik_0 Z_0 \mathbf{J}_\sigma^{kn}(\mathbf{r}')$$

$$+\hat{n}' \times \int_\sigma \mathbf{M}_\sigma^t(\mathbf{r}) \cdot \left[\frac{\nabla \times \underline{\mathbf{\Gamma}}(k_0;\mathbf{r},\mathbf{r}')}{ik_0} - \frac{\nabla \times \underline{\mathbf{\Gamma}}(k_1;\mathbf{r},\mathbf{r}')}{ik_1} \right] dS$$

$$-\hat{n}' \times \int_\sigma \mathbf{J}_\sigma^t(\mathbf{r}) \cdot \left[Z_0 \underline{\mathbf{\Gamma}}(k_0;\mathbf{r},\mathbf{r}') - Z_1 \underline{\mathbf{\Gamma}}(k_1;\mathbf{r},\mathbf{r}') \right] dS$$

$$-Z_1 \hat{n}' \times \int_S \mathbf{J}_S(\mathbf{r}) \cdot \underline{\mathbf{\Gamma}}(k_1;\mathbf{r},\mathbf{r}')\, dS, \quad \mathbf{r}' \in \sigma$$

•FOR OBSERVATION POINTS ON σ

$$
\begin{aligned}
&\frac{i}{2}(k_0 Y_0 + k_1 Y_1)\mathbf{M}_\sigma^t(\mathbf{r}') = ik_0 Y_0 \mathbf{M}_\sigma^{kn}(\mathbf{r}') \\
&- \hat{n}' \times \int_\sigma \mathbf{J}_\sigma^t(\mathbf{r}) \cdot \left[\frac{\nabla \times \underline{\mathbf{\Gamma}}(k_0; \mathbf{r}, \mathbf{r}')}{ik_0} - \frac{\nabla \times \underline{\mathbf{\Gamma}}(k_1; \mathbf{r}, \mathbf{r}')}{ik_1} \right] dS \\
&- \hat{n}' \times \int_\sigma \mathbf{M}_\sigma^t(\mathbf{r}) \cdot \left[Y_0 \underline{\mathbf{\Gamma}}(k_0; \mathbf{r}, \mathbf{r}') - Y_1 \underline{\mathbf{\Gamma}}(k_1; \mathbf{r}, \mathbf{r}') \right] dS \\
&+ \frac{i}{k_1} \hat{n}' \times \int_S \mathbf{J}_S(\mathbf{r}) \cdot \nabla \times \underline{\mathbf{\Gamma}}(k_1; \mathbf{r}, \mathbf{r}')\, dS \quad , \quad \mathbf{r}' \in \sigma
\end{aligned}
$$

IN NON-DYADIC FORM

$$-\frac{1}{2}\mathbf{J}_S(\mathbf{r}') = \frac{iY_1}{k_1}\hat{n}' \times \int_\sigma \left[\mathbf{M}_\sigma^t(\mathbf{r}) \cdot \nabla\nabla G(k_1;\mathbf{r},\mathbf{r}') + k_1{}^2 G(k_1;\mathbf{r},\mathbf{r}')\mathbf{M}_\sigma^t(\mathbf{r})\right] dS$$

$$+\hat{n}' \times \int_\sigma \left[\mathbf{J}_\sigma^t(\mathbf{r}) \times \nabla G(k_1;\mathbf{r},\mathbf{r}')\right] dS - \hat{n}' \times \int_S \mathbf{J}_S(\mathbf{r}) \times \nabla G(k_1;\mathbf{r},\mathbf{r}')\, dS \quad , \quad \mathbf{r}' \in S$$

$$\frac{i}{2}(k_0 Z_0 + k_1 Z_1)\mathbf{J}_\sigma^t(\mathbf{r}') = ik_0 Z_0 \mathbf{J}_\sigma^{kn}(\mathbf{r}')$$

$$+\hat{n}' \times \int_\sigma \mathbf{M}_\sigma^t(\mathbf{r}) \cdot \left[\nabla\nabla G(k_0;\mathbf{r},\mathbf{r}') - \nabla\nabla G(k_1;\mathbf{r},\mathbf{r}')\right] dS$$

$$+\hat{n}' \times \int_\sigma \mathbf{M}_\sigma^t(\mathbf{r})\left[k_0{}^2 G(k_0;\mathbf{r},\mathbf{r}') - k_1{}^2 G(k_1;\mathbf{r},\mathbf{r}')\right] dS$$

$$+\hat{n}' \times \int_\sigma \mathbf{J}_\sigma^t(\mathbf{r}) \times \nabla\left[k_0 Z_0 G(k_0;\mathbf{r},\mathbf{r}') - k_1 Z_1 G(k_1;\mathbf{r},\mathbf{r}')\right] dS$$

$$-ik_1 Z_1 \hat{n}' \times \int_S \mathbf{J}_S(\mathbf{r}) \times \nabla G(k_1;\mathbf{r},\mathbf{r}')\, dS, \quad \mathbf{r}' \in \sigma$$

$$
\begin{aligned}
&\frac{i}{2}(k_0 Y_0 + k_1 Y_1)\mathbf{M}_\sigma^t(\mathbf{r}') = ik_0 Y_0 \mathbf{M}_\sigma^{kn}(\mathbf{r}') \\
&-\hat{n}' \times \int_\sigma \mathbf{J}_\sigma^t(\mathbf{r}) \cdot \left[\nabla\nabla G(k_0;\mathbf{r},\mathbf{r}') - \nabla\nabla G(k_1;\mathbf{r},\mathbf{r}')\right] dS \\
&-\hat{n}' \times \int_\sigma \mathbf{J}_\sigma^t(\mathbf{r}) \left[{k_0}^2 G(k_0;\mathbf{r},\mathbf{r}') - {k_1}^2 G(k_1;\mathbf{r},\mathbf{r}')\right] dS \\
&-i\hat{n}' \times \int_\sigma \mathbf{M}_\sigma^t(\mathbf{r}) \times \nabla\left[k_0 Y_0 G(k_0;\mathbf{r},\mathbf{r}') - k_1 Y_1 G(k_1;\mathbf{r},\mathbf{r}')\right] dS \\
&-\hat{n}' \times \int_S \left\{\mathbf{J}_S(\mathbf{r}) \cdot \nabla\nabla G(k_1;\mathbf{r},\mathbf{r}') + {k_1}^2 G(k_1;\mathbf{r},\mathbf{r}')\mathbf{J}_S(\mathbf{r})\right\} dS \quad , \quad \mathbf{r}' \in \sigma
\end{aligned}
$$

SOME OBSERVATIONS ON THE NUMERICAL IMPLEMENTATION

- NOTE THAT

$$\nabla\nabla\left[G(k_0;\mathbf{r},\mathbf{r}') - G(k_1;\mathbf{r},\mathbf{r}')\right] = \frac{1}{4\pi}\nabla\nabla\left[\frac{e^{ik_1R}}{R} - \frac{e^{ik_0R}}{R}\right]$$

$$= \frac{1}{4\pi}\left[\frac{k_0^{\,2} - k_1^{\,2}}{2R}\left(\underline{\mathbf{I}} - \hat{R}\hat{R}\right) + O\left(R^0\right)\right], \quad R = |\mathbf{r} - \mathbf{r}'| \to 0$$

- THUS, THE ENTIRE TERM ABOVE BEHAVES AS A SIMPLE LAYER POTENTIAL.

- THIS ALLOWS US TO USE A COLLOCATION METHOD (PULSE BASIS AND DELTA TESTING FUNCTIONS).

- ALTERNATIVELY, GLISSON'S BASIS AND TESTING FUNCTIONS MAY BE USED.

CLOSING REMARKS

- ANY OTHER COMBINATION OF THE INTEGRAL EQUATIONS ON σ IS NOT RECOMMENDED
  - SPECIFICALLY, EFIE-TYPE EQUATIONS REQUIRE A HIGH-FREQUENCY APPROXIMATION OF THE CHARGE DENSITY ALSO.
- THE PERFECTLY CONDUCTING (LARGE) PART OF THE SCATTERER CAN BE REPLACED BY ONE SATISFYING AN IBC.
-